Two-electron correlations and the acoustoelectric current through a quantum dot

G. Giavaras
Department of Materials, University of Oxford, Oxford OX1 3PH, United Kingdom

(Received 6 September 2009; revised manuscript received 24 January 2010; published 17 February 2010)

We examine the two-electron correlations in a surface acoustic wave (SAW)-induced quantum dot moving in a pinched-off channel. For a small SAW amplitude and a narrow channel, singlet and triplet states are quasidegenerate and both contribute to the second plateau of the quantized acoustoelectric current. If SAW dots containing two electrons are driven through a small static dot whose confining potential is tuned by a gate pulse, then due to the Pauli principle, the current displays a fractional plateau.

DOI: 10.1103/PhysRevB.81.073302 PACS number(s): 73.23.Hk, 73.50.Rb, 73.63.Kv

Moving quantum dots are defined by the potential minima of a surface acoustic wave (SAW) that propagates along a one-dimensional GaAs/AlGaAs channel.[1] The SAW-induced dots capture electrons from a two-dimensional electron gas (2DEG) and transport them along the channel generating a quantized acoustoelectric current (AEC), $I=nef$, where n is the number of electrons, e is the electric charge, and $f \sim 2.7$ GHz is the SAW frequency.[1]

Due to the importance of the SAW devices for spintronics[2–4] and quantum metrology various models have been employed to explain the AEC.[5–7] The AEC has been calculated for two interacting electrons within a quasistatic approximation for the ground singlet state.[7] In this work we show that for a narrow channel the singlet and triplet states in a SAW dot are quasidegenerate and can both contribute to the AEC through the channel. For a fixed SAW wavelength $\lambda \sim 1$ μm the singlet-triplet splitting is small when the SAW potential amplitude is small for example because of screening and/or a small SAW power. Further, we consider SAW dots containing two electrons that are driven through a small static quantum dot (QD) whose confining potential is tuned by a gate pulse (Fig. 1). As shown below the two electrons in the SAW dots can occupy the low-lying eigenstates with approximately equal probability and because of the Pauli principle the induced AEC through the QD displays a fractional plateau at $I \sim 0.75ef$.

In the experiments a metal split gate is used to induce a pinched-off channel which connects two 2DEG regions that act as source and drain, respectively.[1] The electron dynamics is governed by the time-dependent Schrödinger equation in the direction of the channel with the Hamiltonian

$$H(x_1,x_2,t) = h_1 + h_2 + V_{int}. \tag{1}$$

The Coulomb interaction is $V_{int}=e^2/4\pi e_0 e_r d$, $d=\sqrt{(x_2-x_1)^2+\gamma^2}$, $\gamma=10$ nm, where we assume that the electrons occupy the lowest transverse modes. The single electron Hamiltonian is $h=p_x^2/2m^*+V$, with $V=V_{CP}+V_{SAW}$ the potential energy which is the sum of the channel potential barrier, V_{CP}, due to the split gate and the time-dependent SAW potential, V_{SAW}. In this work $V_{CP}(x)=V_c[\cosh(x/lc)]^{-2}$, where V_c and l_c determine the height and the length of the gate-induced potential barrier in the channel.[7] The SAW potential is (almost) completely screened in the source 2DEG region due to the high electron density and hence transport does not take place in this region. Close to the entrance of the channel the electron density is in principle lower and the combination of the channel and the SAW potentials gives rise to moving dots which capture electrons from the 2DEG and transport them along the channel. To take into account screening effects we model the SAW potential as $V_{SAW}(x,t)=V_W(x)\cos[2\pi(x/\lambda-ft)]$, with the x-dependent amplitude: $V_W(x)=V_s\exp(-\beta|x-x_0|)$, $x \leq x_0$, $V_W(x)=V_s$ otherwise and β reflects the degree of screening. x_0 is defined by $V_{CP}(x_0)-E_F=0$ with E_F the Fermi energy in the source 2DEG region which together with the corresponding sheet electron density, n_e, can be determined from a self-consistent Schrödinger-Poisson solution.[8] Typical values for a standard GaAs/AlGaAs quantum well device are $E_F \sim 15$ meV and $n_e \sim 10^{11}$ cm^{-2} at temperature ~ 1 K. Screening effects in the channel due to the gate electrodes are ignored.[7]

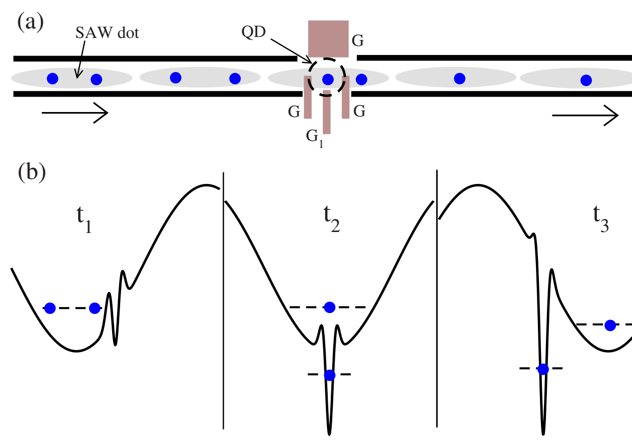

FIG. 1. (Color online) (a) SAW dots carrying two electrons propagate along a channel generating an acoustoelectric current (left-hand side). Electrostatic gates (G_1, G) define a QD whose potential is tuned by applying a pulse to gate G_1. Electrons from the SAW dot tunnel in the QD, with a probability that depends on the symmetry of the two-electron state, and as result the induced acoustoelectric current at the right-hand side decreases. If the tunneling probability is small then both electrons remain in the SAW dot. (b) Potential profile along the channel at three different times ($t_1 < t_2 < t_3$) when a SAW dot which initially contains two electrons interacts with the QD which is initially empty. The depth of the QD increases with time, due to a gate pulse, allowing controlled tunneling from the SAW dot into the QD. The case shown is when one electron from the SAW dot tunnels in the QD as in (a).

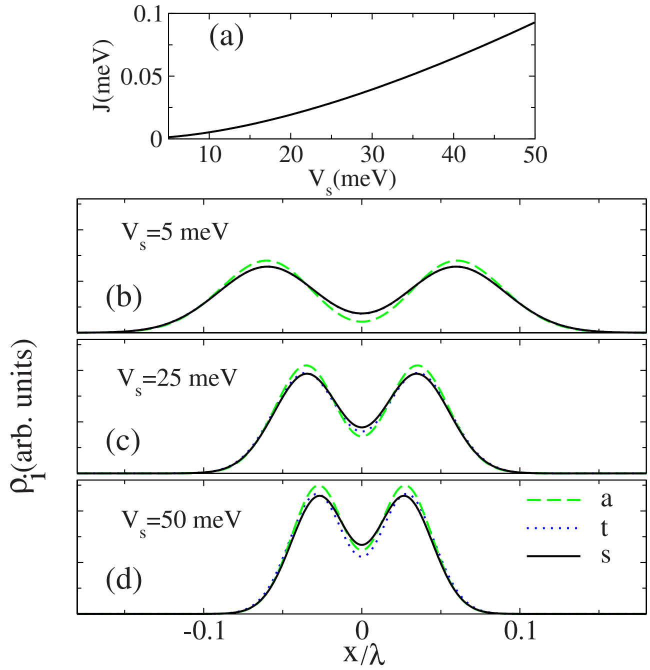

FIG. 2. (Color online) (a) Singlet-triplet energy splitting, J, as a function of the SAW potential amplitude V_s for $\beta=0$. (b)–(d), Two-electron distribution in the SAW dot of singlet, triplet and approximate states ($\rho_i, i=s,t,a$).

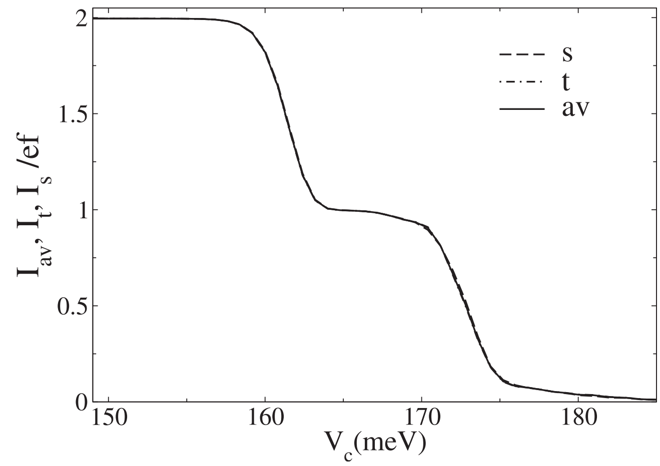

FIG. 3. Acoustoelectric current of singlet (I_s), triplet (I_t) states and average (I_{av}) as a function of the gate-induced barrier height V_c. On this scale the three curves overlap.

First we investigate the SAW dot eigenstates for different SAW amplitudes, V_s, for $V_{CP}=0$, $\beta=0$.[9] For two electrons the spin χ and orbital parts Φ factorize in symmetric and antisymmetric components yielding singlet or triplet states in the form $\Psi_{s/t}=\chi_{s/t}\Phi_{s/t}$. For the diagonalization we use symmetric (singlet) and antisymmetric (triplet) basis states of the form $\Phi_{s/t}=\Sigma_{i\leq j}C_{ij}[\phi_i(x_1)\phi_j(x_2)\pm\phi_j(x_1)\phi_i(x_2)]$, with ϕ_i the single electron SAW dot states. The Coulomb integrals are calculated via a Fourier transform method from the expression $\langle ij|V_{int}|kl\rangle=4\pi\int_0^\infty dqF_{ik}(q)F_{jl}(-q)K(q)$, with $F_{ik}(q)=\frac{1}{2\pi}\int_{-\infty}^{\infty}dx\phi_i(x)\phi_k(x)e^{iqx}$, and the Bessel type function $K(q)=\int dre^{iqr}V_{int}$, $r=x_2-x_1$. The original two-dimensional integrals are transformed to four-dimensional via the Fourier transform and they are subsequently simplified by a standard transform to center of mass and relative motion coordinates.

Figure 2(a) shows the antiferromagnetic exchange energy, i.e., the lowest singlet-triplet splitting $J=E_t-E_s$ and Figs. 2(b)–2(d) show the electron distribution for example of the singlet state $\rho_s(x)=2\int\Phi_s^2(x,x')dx'$. The mean separation between the two electrons decreases with increasing SAW amplitude because the effective width of the SAW dot decreases. As a result the kinetic energy increases with respect to the Coulomb energy and this leads to a large J for a large SAW amplitude. The electrons are close to the strong correlation regime and to a good approximation a Heisenberg Hamiltonian $J\mathbf{S}_1\cdot\mathbf{S}_2$ describes the dynamics which is based on the observation that the orbital distributions of singlet and triplet are almost identical. Another generic signature of this correlated limit is that the first-excited states are separated from the low-lying ones by a large energy gap Δ. For instance, for $V_s=5$ meV we have $\Delta\sim0.5$ meV and $J\sim1$ μeV. This spin Hamiltonian is more accurate in the limit of small SAW amplitudes which can arise because of screening and/or a small applied SAW power. In the source 2DEG region the SAW amplitude increases smoothly as the SAW propagates toward the entrance of the channel, following the change of the local electron density. When a SAW dot is just well defined so that to bind two electrons the SAW amplitude is relatively small yielding a small exchange energy. For temperatures $J\ll k_BT<\Delta$ the SAW dot is likely to transport not only the ground singlet state but also the first-excited triplet states with almost equal probability.

Insight into the electron distribution is provided by a Hartree approximation. Each electron is described by the Hamiltonian $p_{x_i}^2/2m^*+V(x_i)+V_H(x_i)$, $i=1,2$ where the Coulomb interaction is included in an average way by the Hartree term $V_H(x_1)=\int dx_2|\varphi_2(x_2)|^2V_{int}(x_1,x_2)$, with $\varphi_2(x_2)$ the wave function of the lowest (Hartree) energy of the $i=2$ electron. The two one-electron Hamiltonians are coupled and they are solved iteratively. When convergence is achieved the one-electron states are used to form the approximate distribution $\rho_a(x)$ which, as shown in Fig. 2, is in a good agreement with the exact results.

To calculate the AEC we study the time evolution of the orbital parts ($\Phi_{s/t}$) of Eq. (1) whereas the spin eigenstates ($\chi_{s/t}$) remain unchanged. Results are shown in Fig. 3 for various V_c for $V_s=35$ meV and $\beta=1$ μm^{-1}. The current, for example, of the singlet state is $I_s=\Sigma_nP_n^snef$, where $n=0,1,2$ is the number of electrons which are transported by the SAW dot with probability P_n^s. Due to the spin degree of freedom there are three spin states with $S=1$, $S_z=\pm1,0$, for the triplet Φ_t, and one spin state with $S=0$, $S_z=0$ for the singlet Φ_s; therefore, when all states contribute equally the average current is $I_{av}=I_s/4+3I_t/4$.

In Fig. 3 the AEC displays plateaus close to the integer values $n=0,1,2$ a feature which is consistent with the experimental results[1] and an approximate study.[7] In the channel the effective width and depth of the SAW dot are modified due to the presence of the potential barrier V_{CP} and the reduced screening. As V_c decreases the width and depth of the SAW dot increase and hence extra electrons can be transported. For $V_c \sim 165$ meV the SAW dot is in the Coulomb blockade regime transporting a single electron. Beyond this regime there is a finite probability that the second electron will be transported hence the current increases up to the second plateau $V_c \lesssim 158$ meV where both electrons are successfully transported. The accuracy of the plateau depends on the strength of the Coulomb interaction, the probability of backward tunneling from the SAW dot into the source 2DEG region and involves nonadiabatic Landau-Zener transitions.[6]

The difference between singlet and triplet currents is negligible and this is because the tunneling rates of singlet and triplet states are almost identical resulting in a similar dynamics. The other set of parameters can give a somewhat bigger difference but still confirm the fact that the *average* current, which is measured in the experiments, cannot reveal the character of the states that are transported by the SAW dot. Additionally, experimental results have not shown any qualitatively different form for the current from that shown in Fig. 3. However, calculations for a range of model potentials and SAW parameters including a magnetic field are needed to determine whether this condition is met always in these devices. The two components of the current could be probed experimentally when there is a direct signature in the average current and for this to happen singlet and triplet currents have to be different. A way of achieving this is to form a QD in the channel (see Fig. 1) and exploit the different tunnel rates of singlet and triplet. The applied voltage to gate G_1 is tuned in time and as a result the depth of the QD is tuned synchronously enabling electrons to tunnel from the SAW dot to the QD in a controlled fashion.[4] Similar structures have been realized experimentally.[10,11]

The QD is modeled as $V_{QD}(x,t) = V_p(t)\exp(-x^2/2l_p^2)$ and for practical realization the time dependence is controlled by applying a pulse to the gate electrode G_1. $V_p(t) = -V_f(t - t_a)/(t_b - t_a)$, $t_a < t < t_b$ with $t_a = 0.4/f$, $t_b = 0.6/f$, and $V_p(t) = 0(-V_f)$ for $t \leq t_a (t \geq t_b)$. A SAW dot containing two electrons propagates for $0 < t < 1/f$ and the potential is $V = V_{QD} + V_{SAW}$ with $\beta = 0$ in the depleted channel. The AEC for different V_f is shown in Fig. 4. For a small V_f a SAW electron tunnels into the QD and subsequently off due to the SAW propagation therefore $I_{av} \sim 2ef$. Increase in V_f enables single electron tunneling from the SAW dot to the QD up to the point where the first plateau is formed ($V_f \sim 20$ meV) and the QD is in the Coulomb blockade regime. Further increase of $V_f \sim 35$ meV allows both electrons to tunnel from the SAW dot to the QD only for the singlet, whereas for the triplets this event is negligible. This is due to the Pauli exclusion principle which yields much lower energy for the ground singlet in the QD than the triplet and thus higher tunneling probability. The QD has to be small ($l_p = 12$ nm) so when the two electrons are bound to be weakly correlated. In this regime $I_s \sim 0$ and $I_t \sim ef$ hence the average current displays a plateau at $I_{av} \sim 0.75ef$. When $V_f \sim 42$ meV both singlet and triplet states tunnel in the QD and $I_{av} \sim 0$.

After a SAW dot has passed the QD then the latter is typically loaded with electrons. To allow repetition of the cycle the QD has to be reset (unloaded and tuned to the initial potential profile[12]) before the next SAW dot that follows interacts with the QD. It might be difficult to reset the QD on a time scale of the SAW period ~ 0.4 ns, so it might

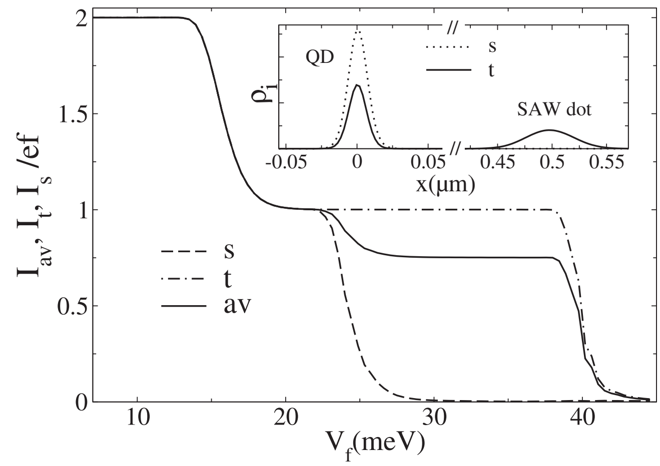

FIG. 4. Acoustoelectric current of singlet (I_s), triplet (I_t) states and average (I_{av}) when the SAW dots are driven through a quantum dot whose potential depth is V_f. (inset) Singlet and triplet distributions ($\rho_i, i = s, t$, in arb. units) for $V_f = 33$ meV.

be useful to adjust the SAW to carry two electrons at every N_m potential minima.[3] $N_m = 10$ allows the QD to be reset within ~ 4 ns and still produces a large measurable AEC on the order of $2ef/N_m \sim 86$ pA.

The electron distribution in Fig. 4 (inset) suggests that the dynamics of the continuum model [Eq. (1)] can be mapped to an effective lattice model. A three-site Hubbard Hamiltonian captures all the important physics,

$$\mathcal{H}(t) = \sum_{i=1}^{3} \varepsilon_i(t) n_i - \gamma \sum_{\sigma,i=1}^{2} (c_{i\sigma}^\dagger c_{i+1\sigma} + \text{H.c.}) + U \sum_{i=1}^{3} n_{i\uparrow} n_{i\downarrow} + V \sum_{i=1}^{2} n_i n_{i+1}, \tag{2}$$

with $\sigma = \uparrow, \downarrow$ and $n_i = \Sigma_\sigma n_{i\sigma} = \Sigma_\sigma c_{i\sigma}^\dagger c_{i\sigma}$. $c_{i\sigma}^\dagger(c_{i\sigma})$ creates (destroys) an electron on site i with spin σ. U, V, γ change in time but we assume that only the on-site energies, $\varepsilon_i(t)$, are time dependent. Still with this assumption the model captures the basic mechanism of the $0.75ef$ plateau and offers a simple explanation. The significance of each site is as follows: at the initial time, $t = 0$, $i = 1$ and 2 correspond to the positions of the two electrons in the SAW and $i = 3$ corresponds to the front SAW potential maximum. ε_2 decreases for $0 < t < \tau_{QD}$ reflecting the formation of the QD, while $\varepsilon_1(\varepsilon_3)$ increases (decreases) for $0 < t < \tau_{SAW}$ due to the SAW propagation. At the final time, $t = \tau_{SAW}$, $i = 1(3)$ models the rear SAW potential maximum (minimum) and $i = 2$ the QD. We consider a sinusoidal time dependence for ε_1, ε_3 and a linear for ε_2 as well as $\tau_{QD} < \tau_{SAW}$ consistent with the continuum model.[13]

For two electrons and zero magnetic field there are six singlet and three triplet basis states and to demonstrate the dynamics that yields the $0.75ef$ plateau we consider the regime $U \gg \gamma$, $V = U/5$. At the initial time and for $\varepsilon_3(0) \gg \varepsilon_{1,2}(0)$ the SAW electrons are described by eigenstates of Eq. (2) with the approximate form $(c_{1\uparrow}^\dagger c_{2\downarrow}^\dagger \pm c_{1\downarrow}^\dagger c_{2\uparrow}^\dagger)|0\rangle/\sqrt{2}$ with $-(+)$ for singlet ($S_z = 0$ triplet) and $|0\rangle$ the vacuum state.

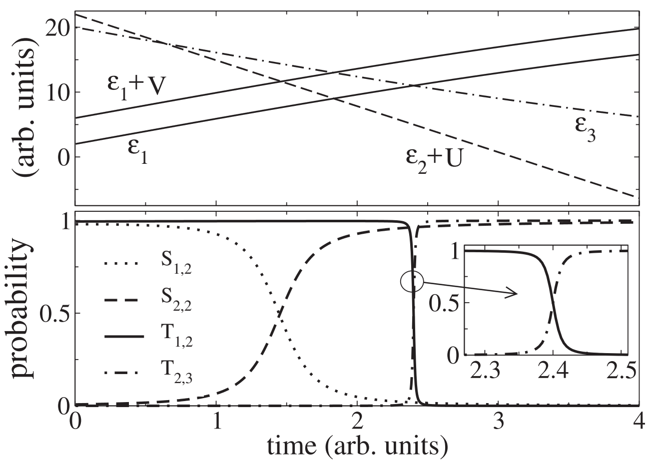

FIG. 5. (top) Energy scales involved in the Hubbard model. (bottom) Singlet ($S_{i,j}$) and triplet ($T_{i,j}$) probabilities in the regime of the $0.75ef$ plateau.

Figure 5 shows the occupation probabilities with an appreciable magnitude when both τ_{QD} and τ_{SAW} are chosen long enough to ensure approximately adiabatic evolution as in the continuum model. At the final time $T_{2,3} \sim 1$, $S_{2,2} \sim 1$ which means that the triplet state $(c_{2\uparrow}^{\dagger}c_{3\downarrow}^{\dagger}+c_{2\downarrow}^{\dagger}c_{3\uparrow}^{\dagger})|0\rangle/\sqrt{2}$ and the singlet $c_{2\uparrow}^{\dagger}c_{2\downarrow}^{\dagger}|0\rangle$ respectively are (almost) fully occupied in agreement with the electron distribution in Fig. 4. Within the single orbital Hubbard model doubly occupied sites are allowed only for the singlet due to the Pauli principle. The initial singlet state changes within the time interval for which the energy condition $\varepsilon_1+V \sim \varepsilon_2+U$ is satisfied, that to a good approximation allows resonant tunneling of an electron from $i=1$ to $i=2$ when a second electron is already on $i=2$. Similarly, the resonance condition for the triplet state is $\varepsilon_1 \sim \varepsilon_3$ which leads to the sharp transition as seen in Fig. 5. First an electron tunnels from $i=2$ to $i=3$ and this process is followed by the second electron that tunnels from $i=1$ to $i=2$ to yield the final triplet state.

Fractional plateaus ($\sim 0.6ef$) have been observed experimentally without however a clear explanation.[10] Our calculations indicate that their origin might be related to spin effects and therefore the application of a magnetic field could give further insight into the physics and could even reveal extra plateaus. The exact value of the plateaus depends on the temperature and this could be probed for typical experimental temperatures in the range $\sim 0.1-1$ K.

In summary, both singlet and triplet states can contribute to the second plateau of the AEC. This is more likely to occur when the two electrons in the SAW dots are in the strong correlation regime with a small antiferromagnetic exchange energy. If SAW dots carrying two electrons are driven through a QD that is tuned by a gate pulse then the AEC displays a fractional plateau at $\sim 0.75ef$. This spin effect is a consequence of the Pauli principle and our calculations indicate that it could be observed.

[1] J. M. Shilton, V. I. Talyanskii, M. Pepper, D. A. Ritchie, J. E. F. Frost, C. J. B. Ford, C. G. Smith, and G. A. C. Jones, J. Phys.: Condens. Matter **8**, L531 (1996).

[2] G. Giavaras, J. H. Jefferson, A. Ramsak, T. P. Spiller, and C. J. Lambert, Phys. Rev. B **74**, 195341 (2006).

[3] C. H. W. Barnes, J. M. Shilton, and A. M. Robinson, Phys. Rev. B **62**, 8410 (2000).

[4] G. Giavaras, J. H. Jefferson, M. Fearn, and C. J. Lambert, Phys. Rev. B **76**, 245328 (2007).

[5] A. M. Robinson and C. H. W. Barnes, Phys. Rev. B **63**, 165418 (2001).

[6] P. A. Maksym, Phys. Rev. B **61**, 4727 (2000).

[7] Godfrey Gumbs, G. R. Aizin, and M. Pepper, Phys. Rev. B **60**, R13954 (1999).

[8] G. Giavaras, Semicond. Sci. Technol. **23**, 085010 (2008).

[9] Only for the dynamics we consider a fixed V_s, $\beta \neq 0$.

[10] J. Ebbecke, N. E. Fletcher, T. J. B. M. Janssen, H. E. Beere, D. A. Ritchie, and M. Pepper, Phys. Rev. B **72**, 121311(R) (2005), and references therein.

[11] M. Kataoka, R. J. Schneble, A. L. Thorn, C. H. W. Barnes, C. J. B. Ford, D. Anderson, G. A. C. Jones, I. Farrer, D. A. Ritchie, and M. Pepper, Phys. Rev. Lett. **98**, 046801 (2007).

[12] The electrons in the QD can be removed via tunneling in the adjacent 2DEG by tuning the voltage to gate G_1.

[13] Due to the SAW propagation for $t > \tau_{QD}$ the QD potential is shifted upwards in energy and tunneling from the QD to the SAW dot can occur. However, the electron distribution in Fig. 4 indicates that this process is weak and therefore it has been ignored in the Hubbard model.